Bayesian Methods for Multi-Objective Optimization of a Supersonic Wing Planform

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Contents

- Introduction to SSTs
- Study aims
- Analysis geometry
- Workflow walk-through
- Results

Concorde in flight.
Photo: Eduard Marmet, 1986

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Supersonic Transports 1

- Revival of interest in commercial SSTs
- Aircraft design has many trade-offs
- Airlines want:
  - Low running cost and greater capacity
- Passengers want:
  - Speed and comfort at a low cost
- People on the ground want:
  - Less noise, less environmental impact
- → Design for greater lift to drag & less noise

Aerion AS2 (old design)
(Original: https://www.aerionsupersonic.com/)

Boom Overture
(Original: http://boomsupersonic.com/)

Lockheed Martin X-59 QueSST
(Original: https://nasa.gov)

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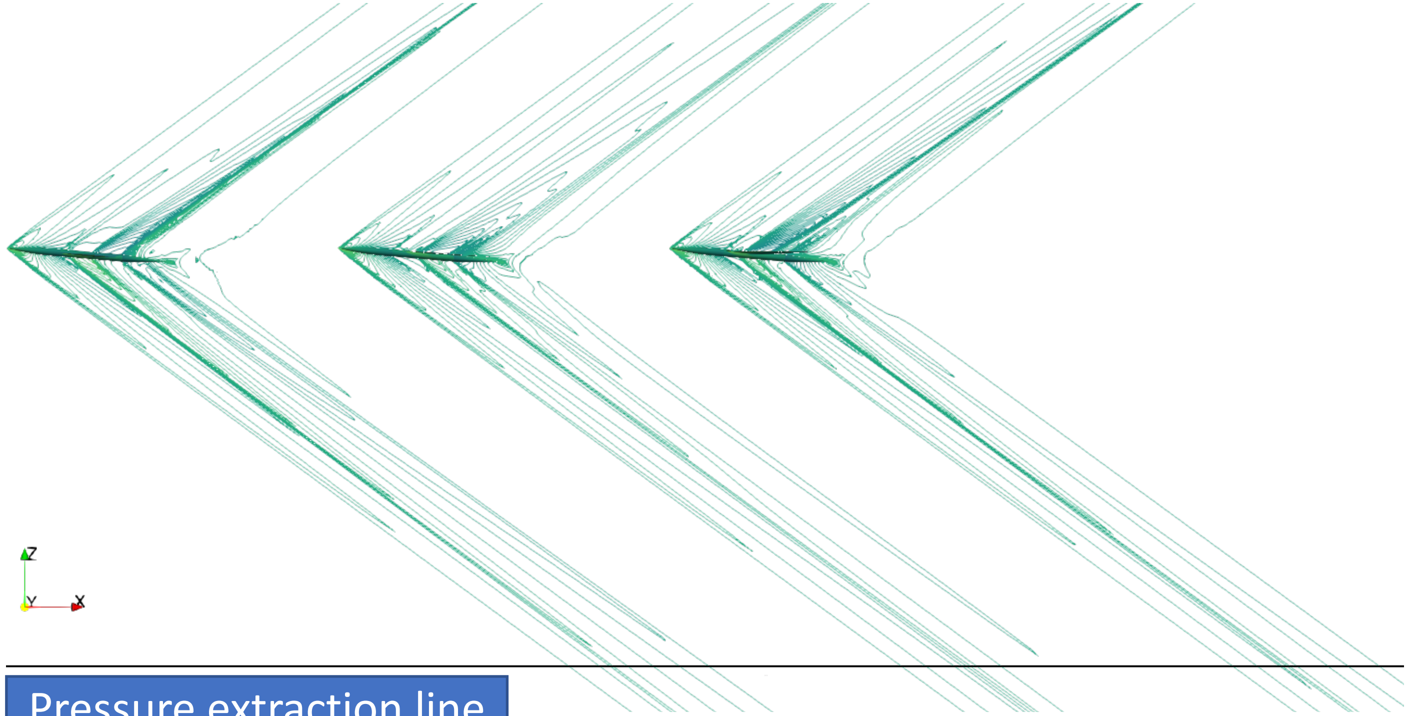
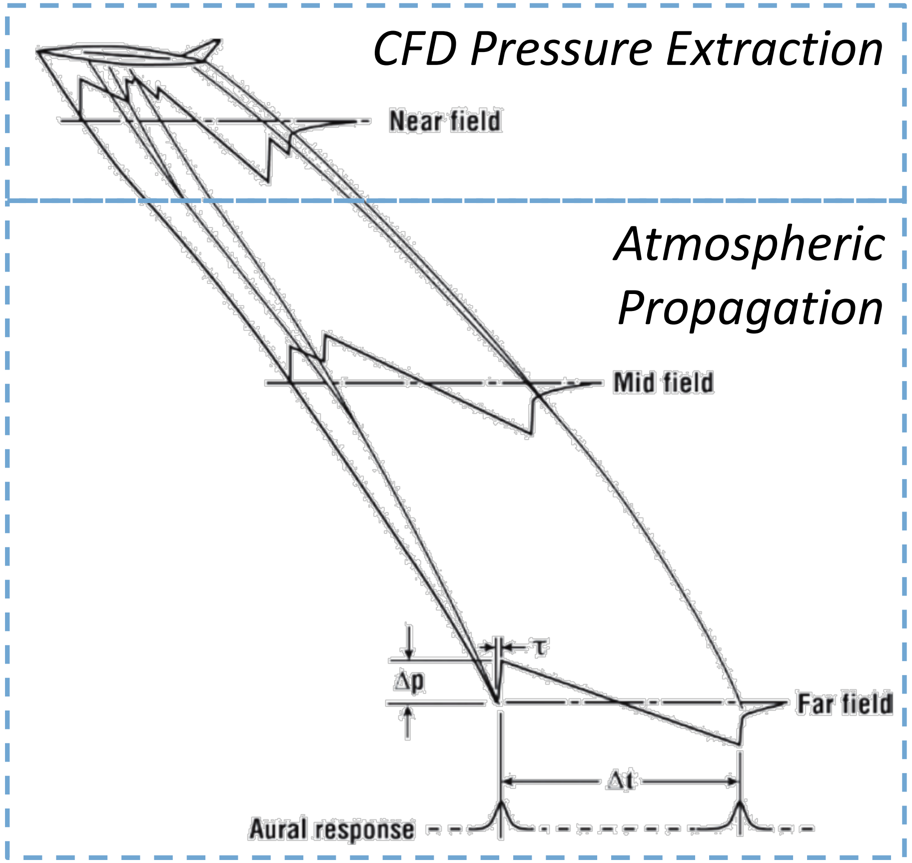
Supersonic Transports 2

- Supersonic flight creates shockwaves
  - The shocks are propagated to the ground
- Performance and noise affected by design!



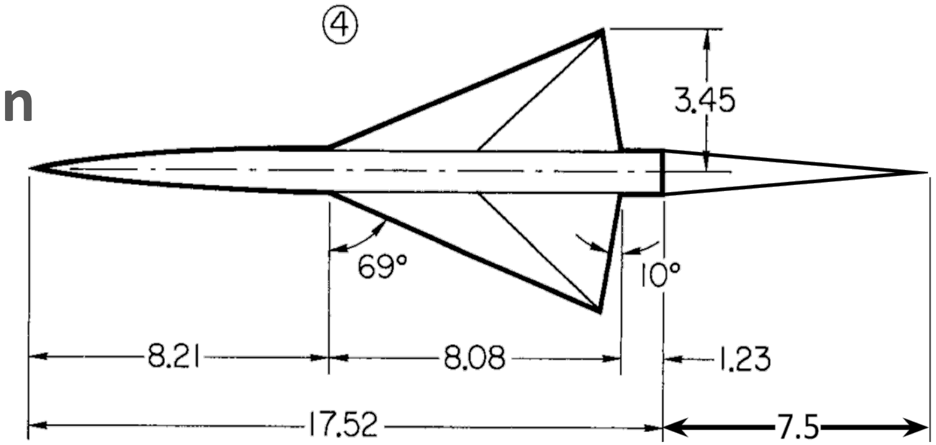
Pressure extraction line



Ground level noise prediction process
(Original: Plotkin, 1989)

Study Aims

- Optimise: "Realistic" SST by wing planform (sectional-shape) design
  - Low drag (inviscid)
  - Low ground-level noise (A-weighted)
  - Subject to: minimum lift constraint and geometric constraints
- Using: EHVI-Kriging Believer-based BO framework
- Compare the effectiveness of solvers
  - Gradient-based vs GA solver for **EHVI optimisation**
  - 6-var problem vs 11-var problem

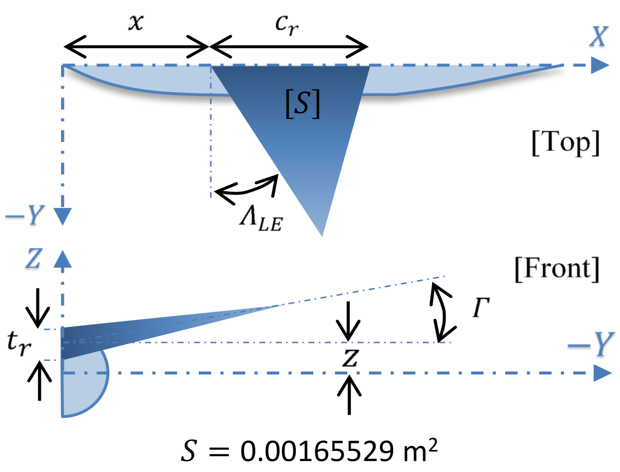


Modified 69-degree Delta-Wing Body
(Original: Hunton, 1973)

Parameterisation Models

1-Section Wing (6 Var.)

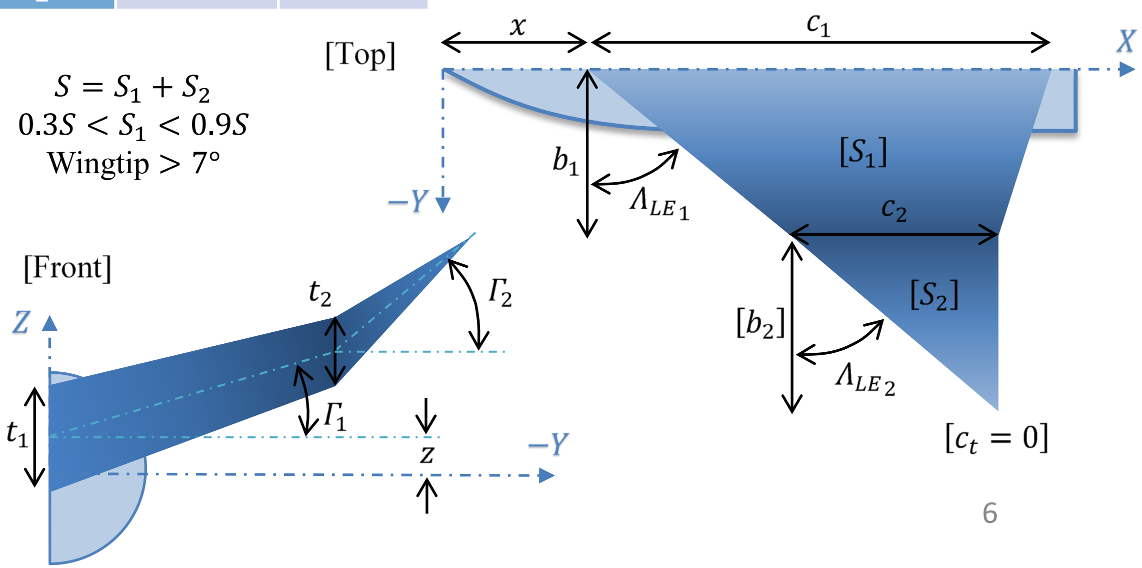
Symbol	Parameter	Min	Max
x	x-location of wing (m)	0.05	0.1
z	z-location of wing (m)	-0.0045	0.0045
Λ_{LE}	leading edge sweep (°)	-65	75
Γ	dihedral (°)	-15	15
c_r	root chord (m)	0.04	0.11
t_r	root thickness / chord	0.02	0.1



2-Section Wing (11 Var.)

Symbol	Min	Max
x	0.05	0.1
z	-0.0045	0.0045
$\Lambda_{LE\,1}$	-65	75
Γ_1	-30	30
c_1	0.03	0.11
t_1	0.02	0.1
b_1	0.01	0.06

Symbol	Min	Max
$\Lambda_{LE\,2}$	-65	75
Γ_2	-45	45
c_2	0.021	0.11
t_2	0.02	0.1



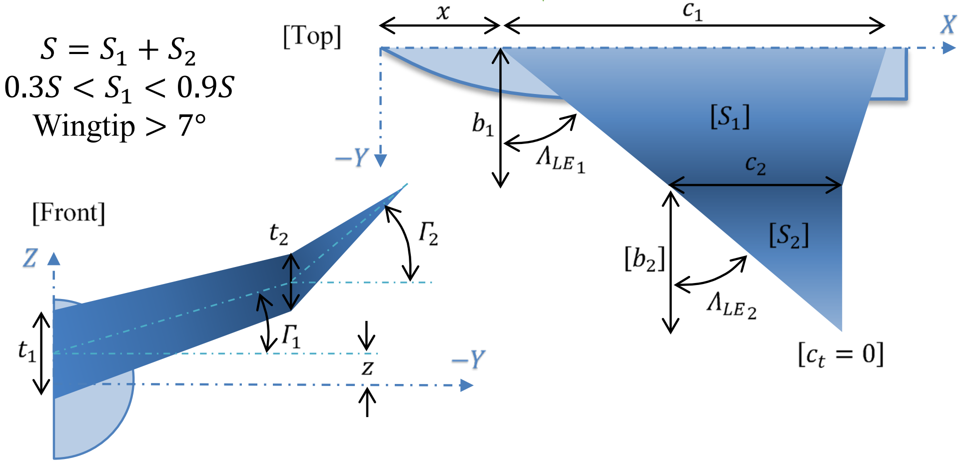
N.B. All wings use a diamond cross-section.

Objectives, Constraints, and Settings

Objectives and Constraints

Objective 1	Minimize inviscid drag (C_D)
Objective 2	Minimize A-weighted ground-level noise (dBA)
Constraint 1 *("expensive")*	Lift coefficient ($C_L \geq 0.15$)
Constraints 2, 3 *("cheap")*	Geometric constraints

$S = S_1 + S_2$
$0.3S < S_1 < 0.9S$
Wingtip $> 7°$



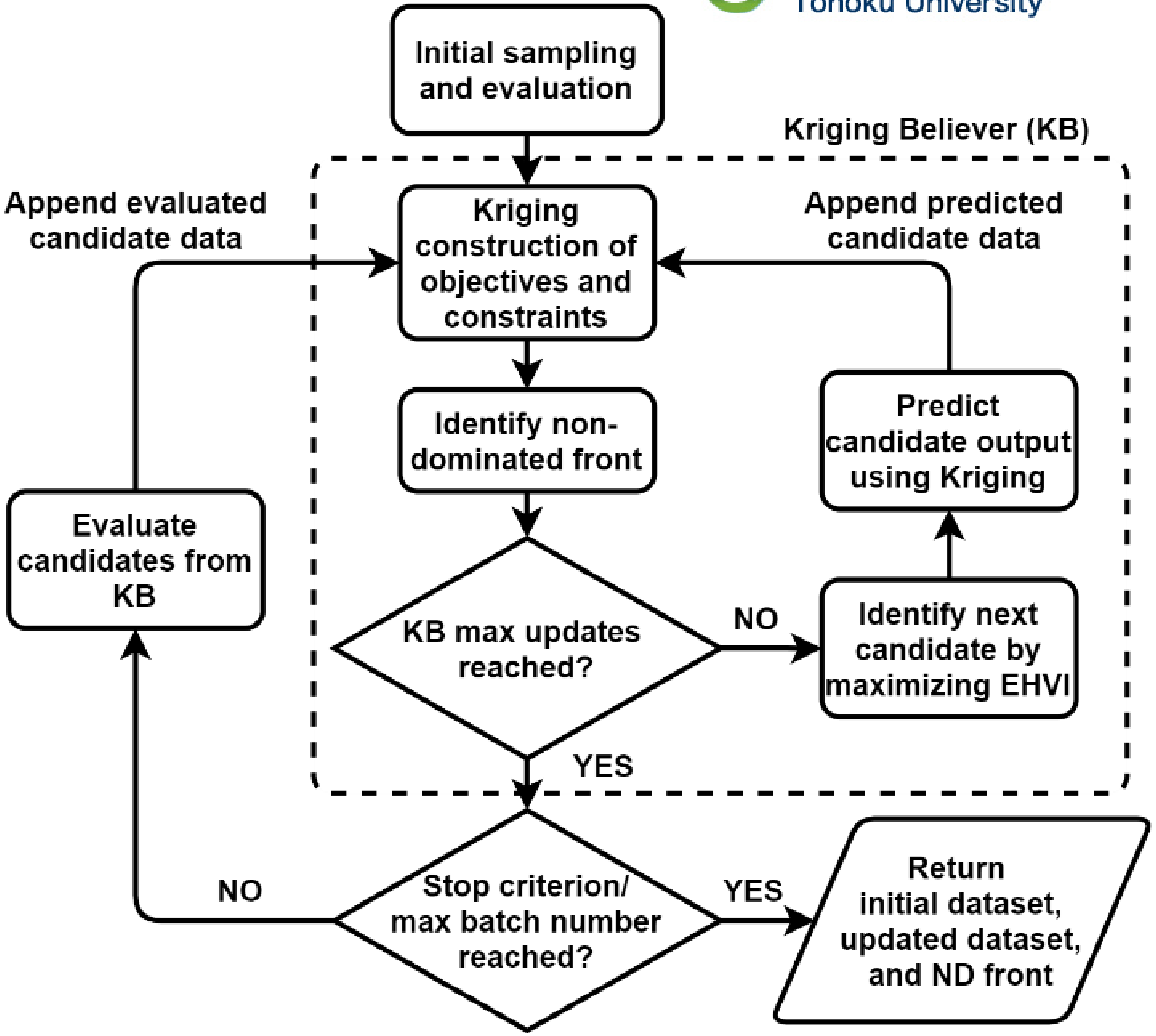
CFD (SU2)

Angle of attack (α)	4.9°
Mach number (M)	1.7
Flight altitude	55,000 ft (Std. Atm.)

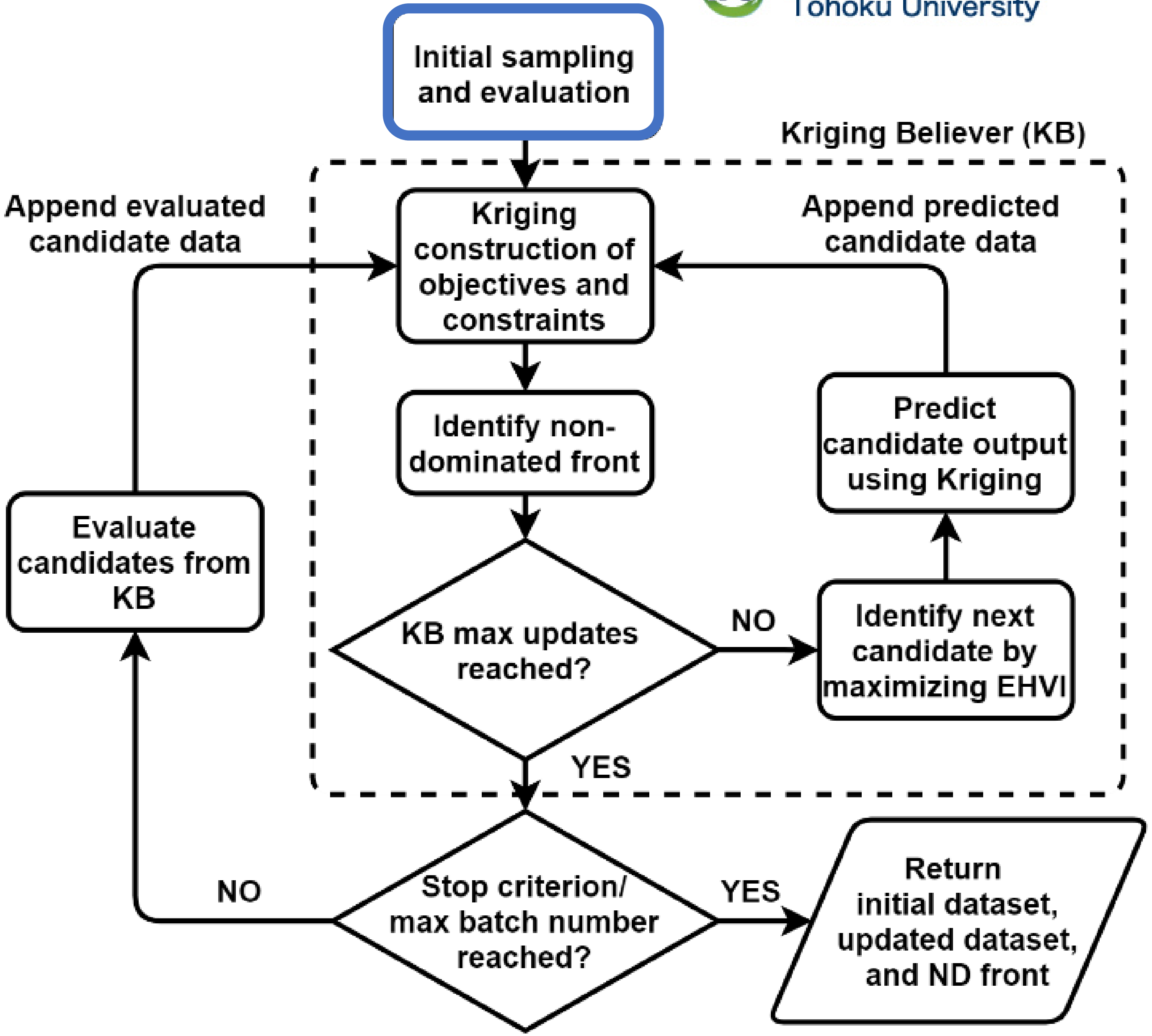
Atmospheric Boom Propagation (NASA sBOOM)

Pressure extraction radius	~2.5 body-lengths
Wave discretisation	3000 points
Extraction angles	0°
Reflection factor	1.9
Model scale	0.0065

Optimization Workflow

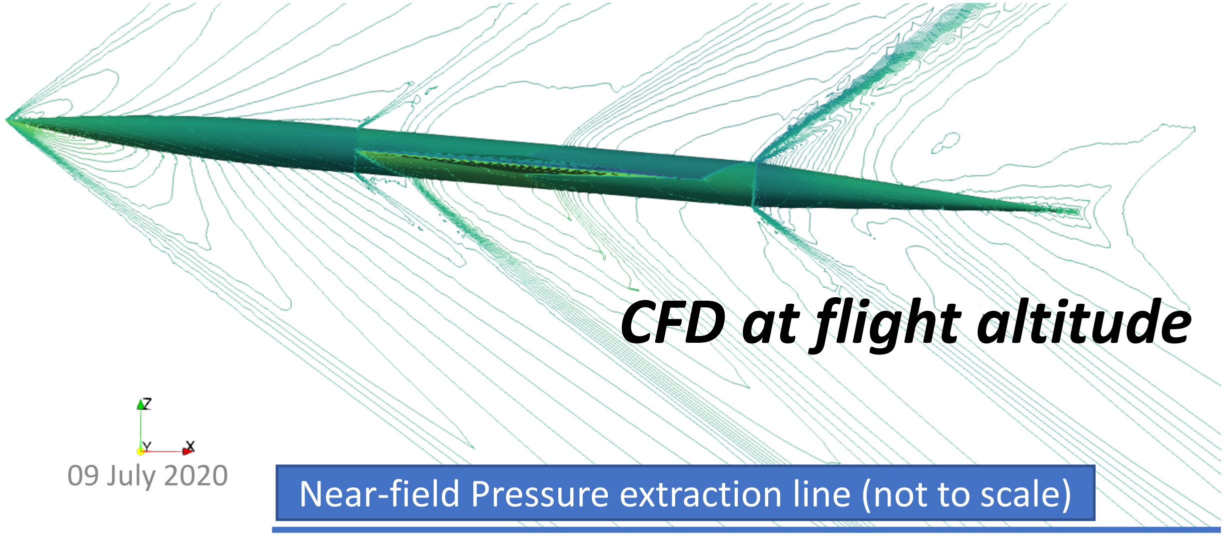


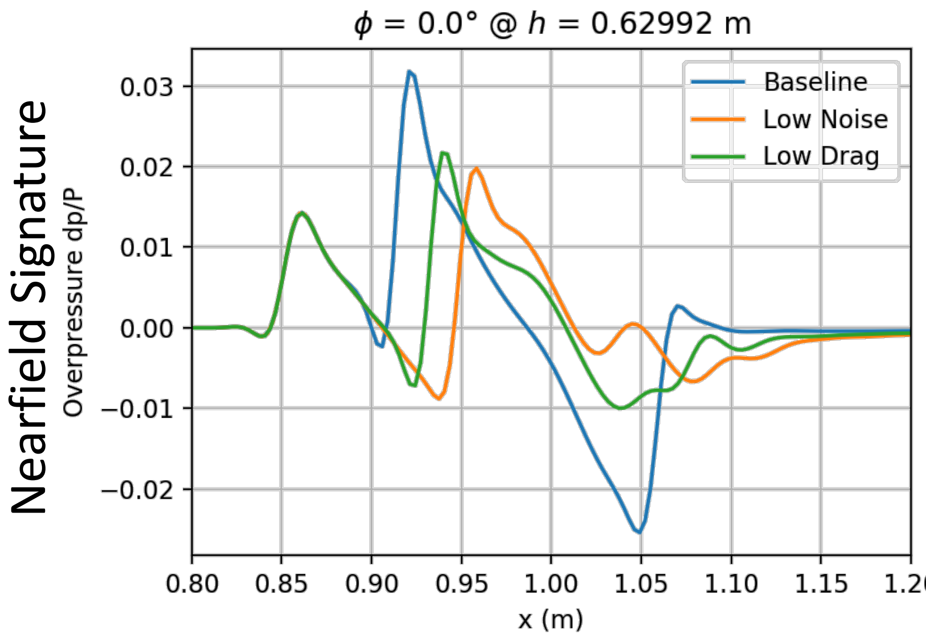
Initial Sampling and Evaluation

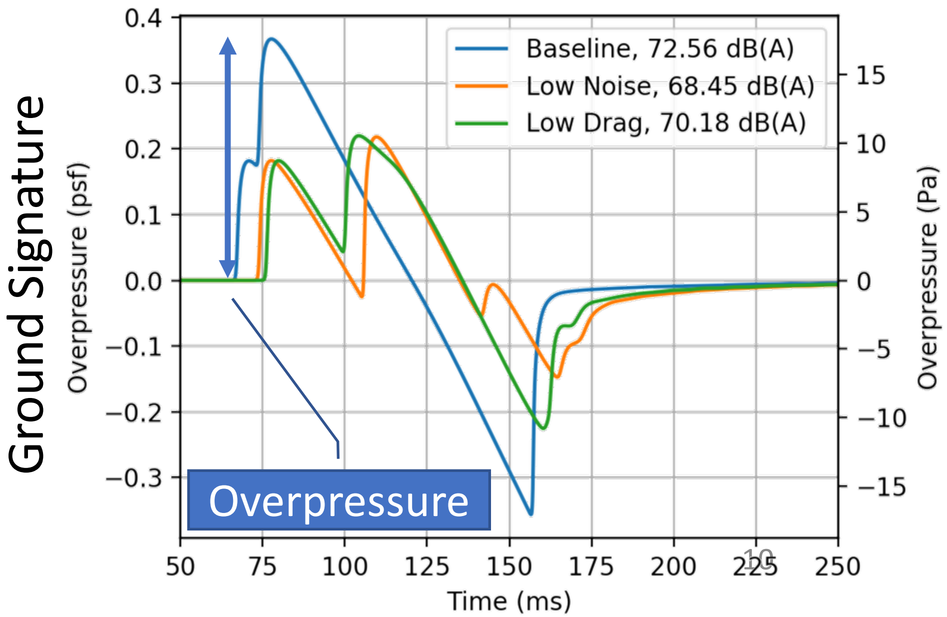


Lift, Drag, and Noise

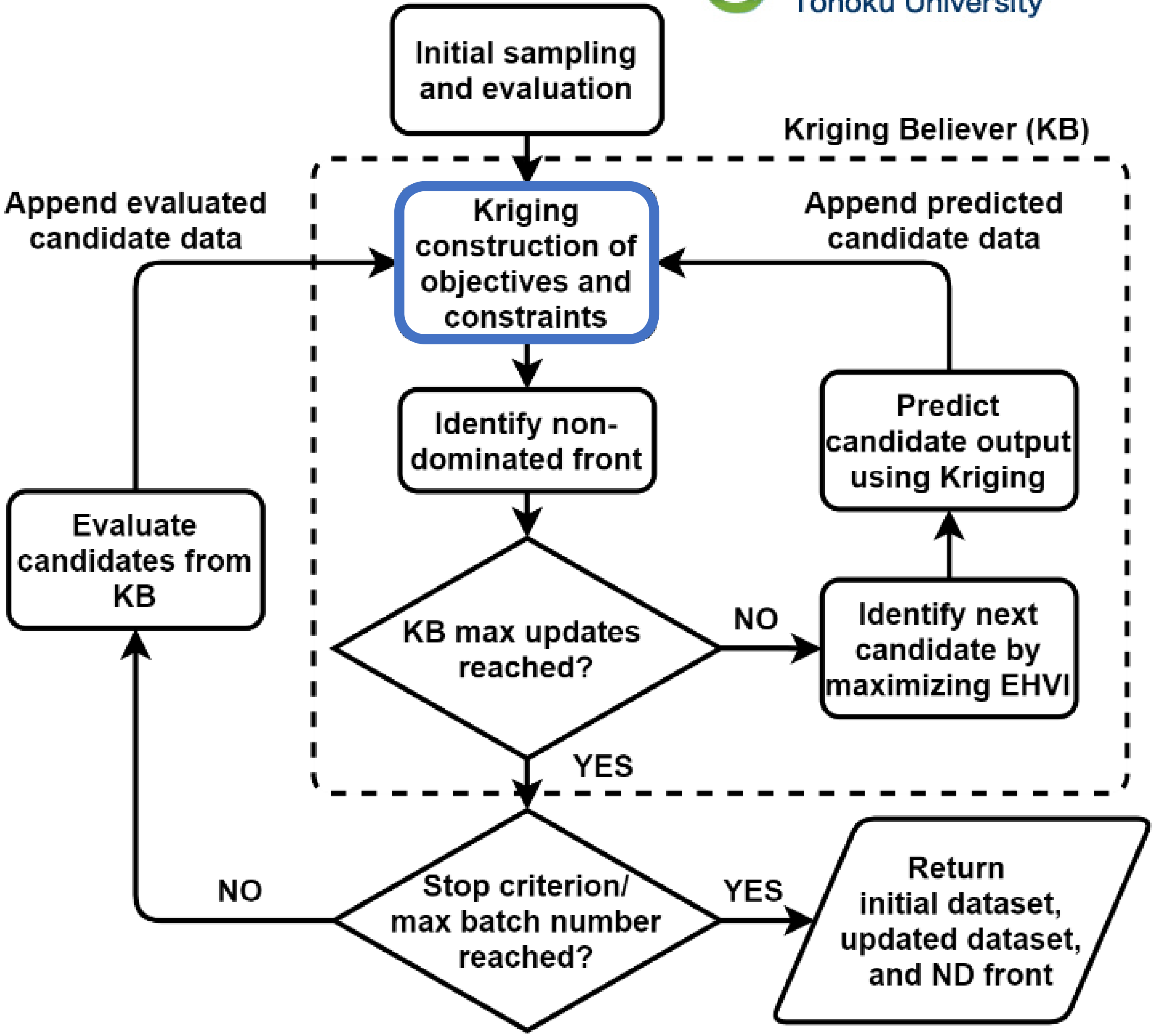
- CFD evaluates the aircraft near-field (SU2)
  - Lift, inviscid drag, near-field pressure
- Atmospheric propagation of near-field pressure to the far-field (NASA sBOOM)
  - Noise (dBA) evaluated from far-field pressure
  - Assuming no wind or turbulence



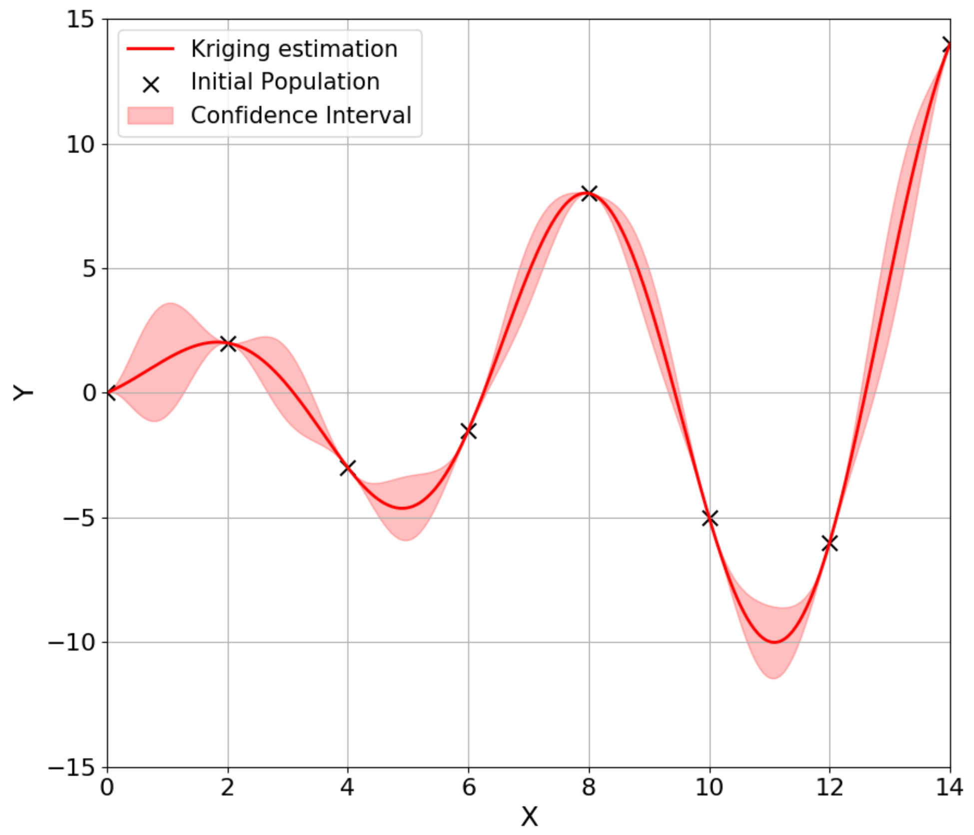




Gaussian Process



Kriging (Gaussian Process) Model



- GP models predict a system's outcome
  - Model relationship between:
    - Input variables $\boldsymbol{X} = \{\boldsymbol{x}^{(1)}, \dots, \boldsymbol{x}^{(n)}\}$ and
    - Response $\boldsymbol{y} = \{y^{(1)}, \dots, y^{(n)}\}$
    - Using Gaussian autocorrelation
  - Hyperparameters $\boldsymbol{\theta}$ are optimized
    - Maximising the ln-likelihood function
    - $\boldsymbol{\theta} = \{\theta_1, \dots, \theta_m\}$, represent variable length scales
  - Produce a **prediction** and its **uncertainty**
- Useful in the place of expensive evaluations
  - CFD!

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Kriging (Gaussian Process) Model

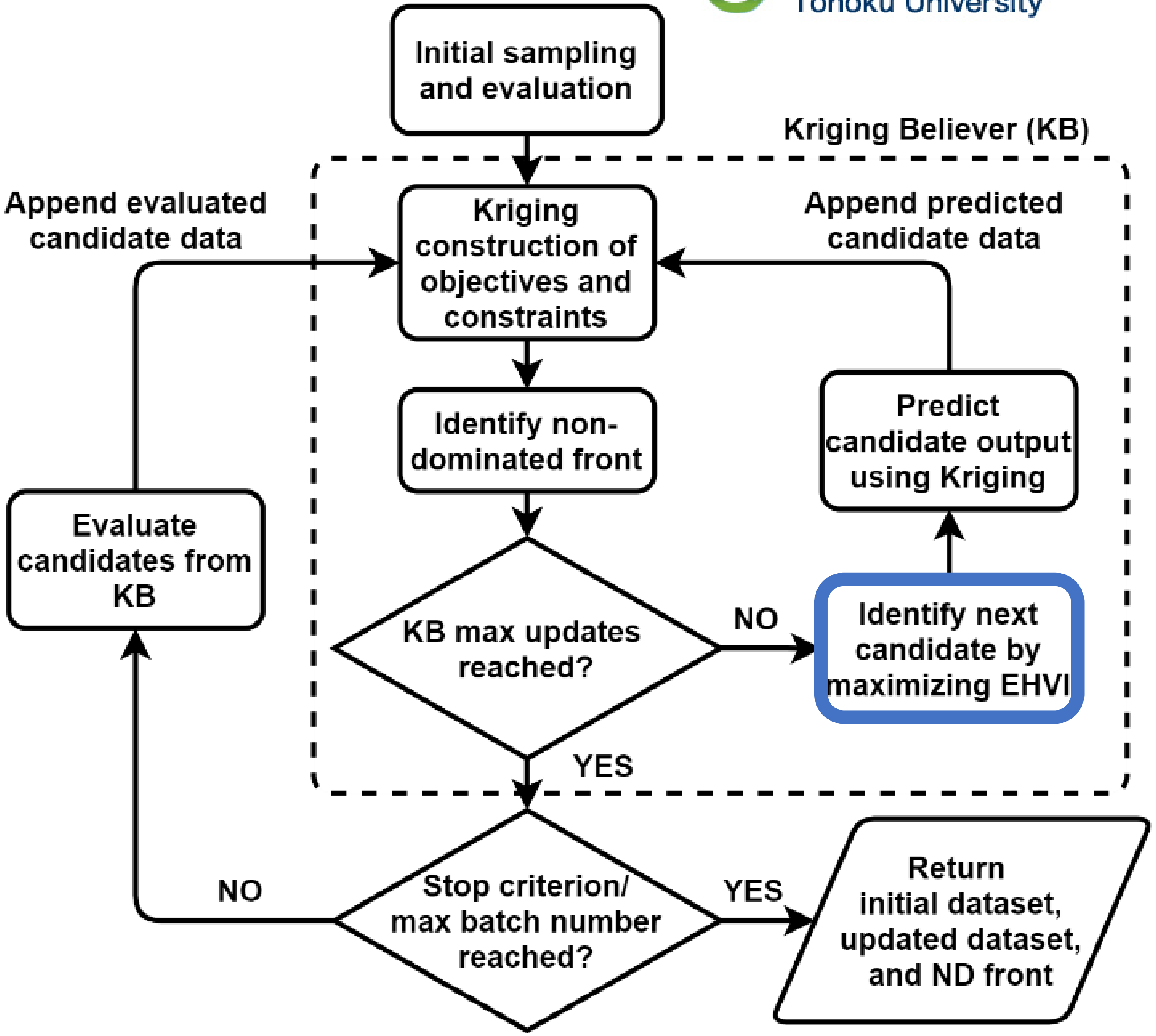
- GP models predict system's outcome by modeling relationship between system input variables $\boldsymbol{X} = \{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(n)}\}$ and its corresponding response $\boldsymbol{y} = \{y^{(1)}, \ldots, y^{(n)}\}$ using Gaussian autocorrelation:

$$cor\left[y\left(\boldsymbol{x}^{(i)}\right), y\left(\boldsymbol{x}^{(l)}\right)\right] = \exp\left(-0.5 \sum_{j=1}^{m} \left(\frac{x_j^{(i)} - x_j^{(l)}}{\theta_j}\right)^{\boldsymbol{2}}\right)$$

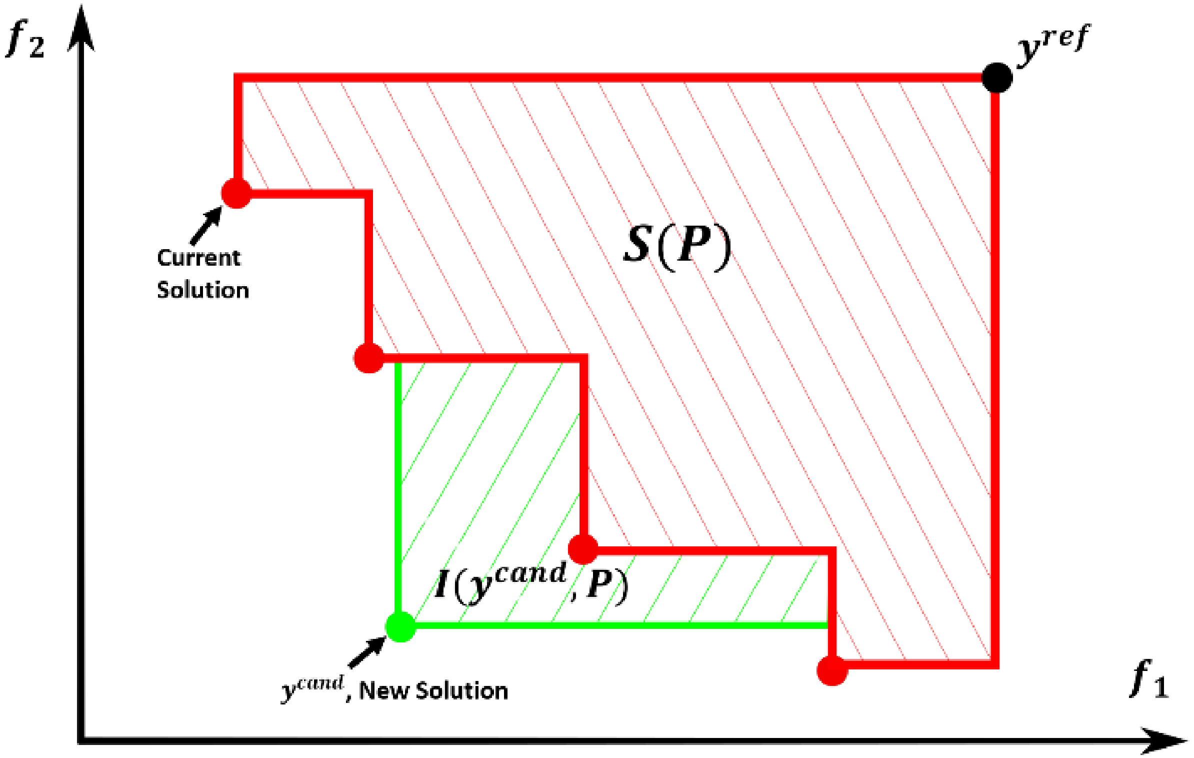
- Hyperparameters $\boldsymbol{\theta} = \{\theta_1, \ldots, \theta_m\}$**,** which represents the length scale of each variable dimension that needs to be optimized by maximizing ln-likelihood function to obtain an accurate Kriging model.
- In our case, we use a gradient-based optimizer with 5 different starting points to optimize the hyperparameter.
- Prediction at the unobserved input $\boldsymbol{x}^*$ can be estimated using the following equation

$$\hat{y}(\boldsymbol{x}^*) = \hat{\mu} + \boldsymbol{\psi}^T \boldsymbol{\Psi}^{-1}(\boldsymbol{y} - \boldsymbol{1}\hat{\mu})$$

Maximising EHVI
(Gradient vs GA)



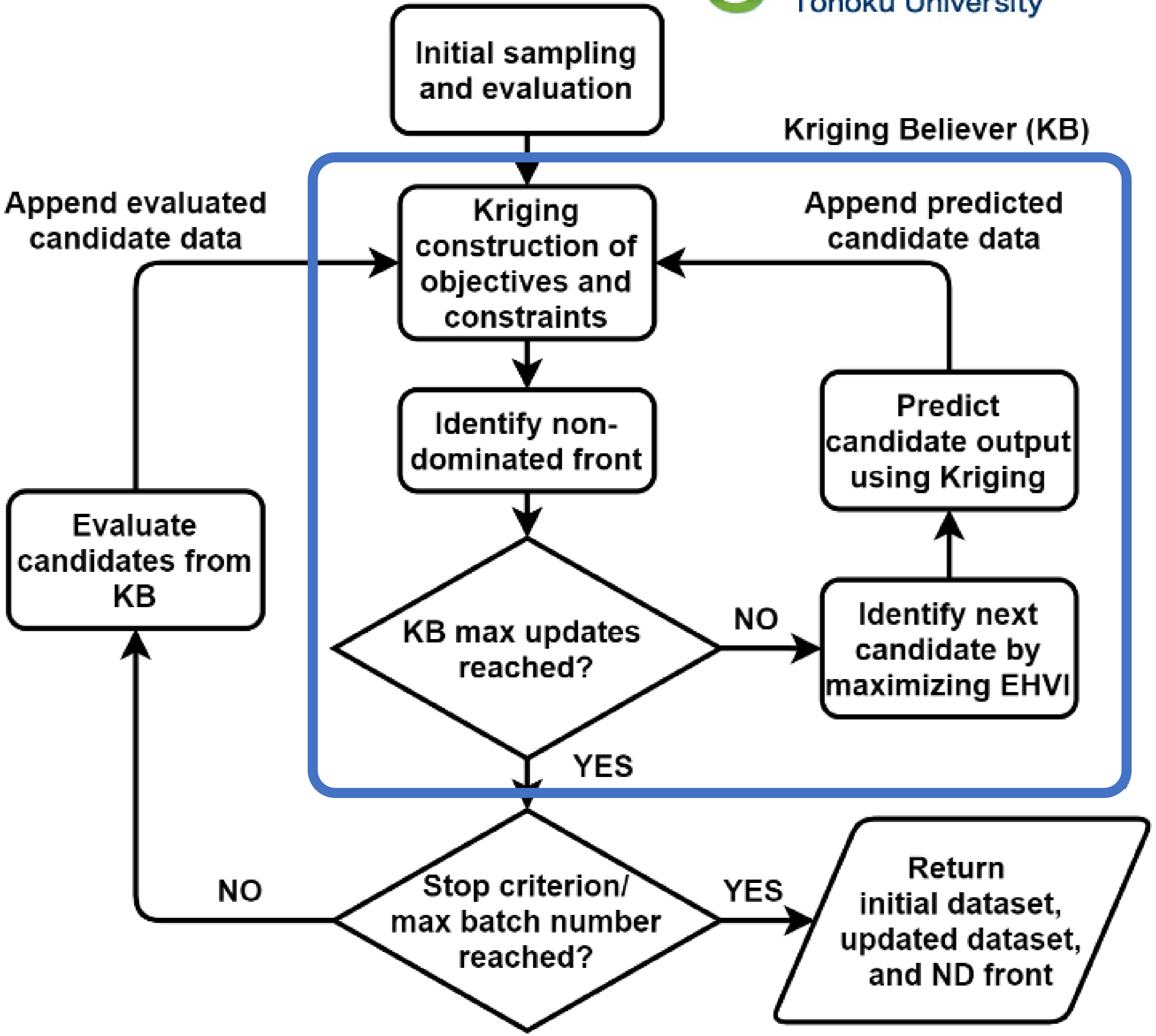
Constrained EHVI



- EHVI formulation:
  - $\mathrm{EHVI}(\boldsymbol{x}) = \int_{\vec{y} \in V_{nd}} I(\vec{y}, P) \cdot \mathrm{PDF}_{\mathbf{x}}(\vec{y}) d\vec{y}$
  - (Emmerich et.al, 2008)
- As we deal with both expensive and cheap constraint, EHVI value is modified:
  - $\mathrm{EHVI}_{\mathrm{constrained}}(x) = \mathrm{EHVI}(x) \cdot \prod_{i=1}^{n\_const} \mathrm{PoF}_i(x)$
- Probability of Feasibility (PoF) for cheap constraints is 1 (feasible) or 0 (violated)
- For expensive constraints
  - $\mathrm{PoF}(\boldsymbol{x}^*) = \frac{1}{\hat{s}\sqrt{2\pi}} \int_0^{\infty} \exp\left(\frac{-(F-\hat{g}(x^*))^2}{2\,\hat{s}^2}\right) dG$
  - (Forrester, Sóbester, Keane, 2008)

For a more-detailed explanation, see:
Shimoyama, Jeong, Obayashi, *2013*, Kriging-surrogate-based optimization considering expected hypervolume improvement in non-constrained many-objective test problems

Kriging Believer Loop



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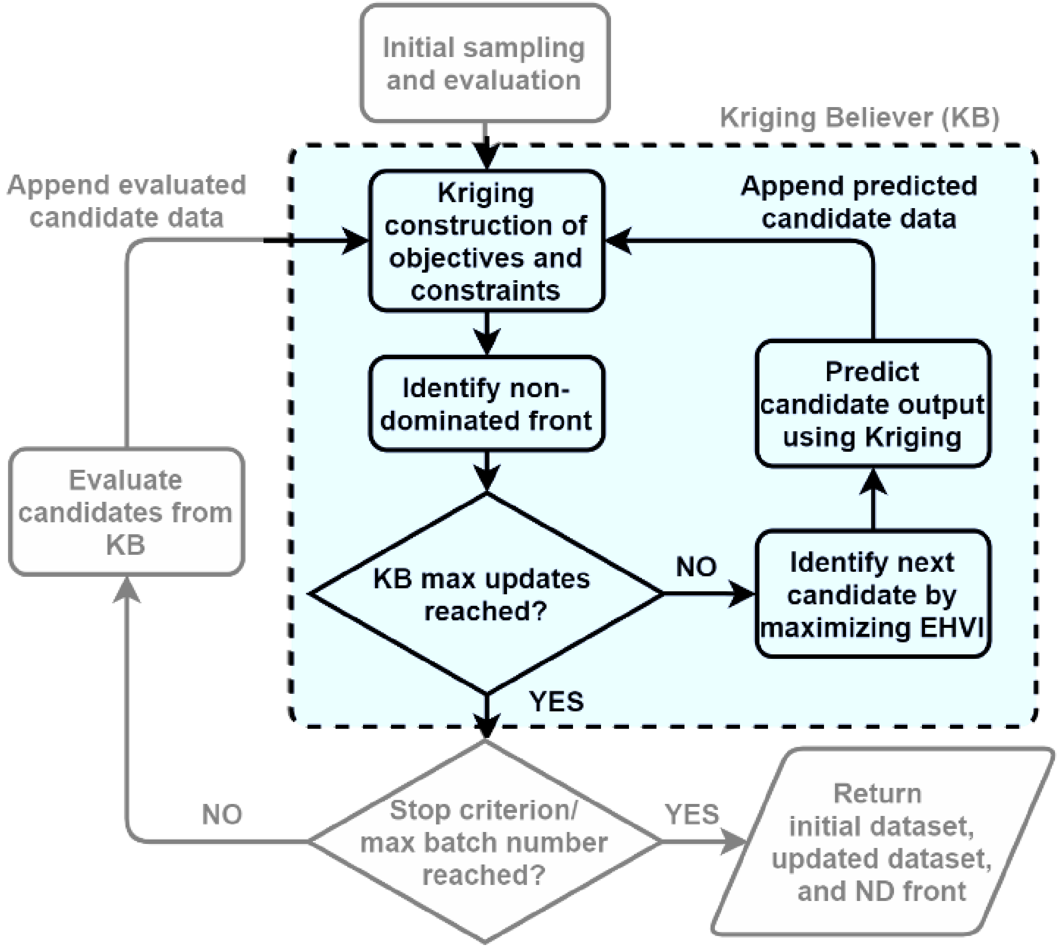
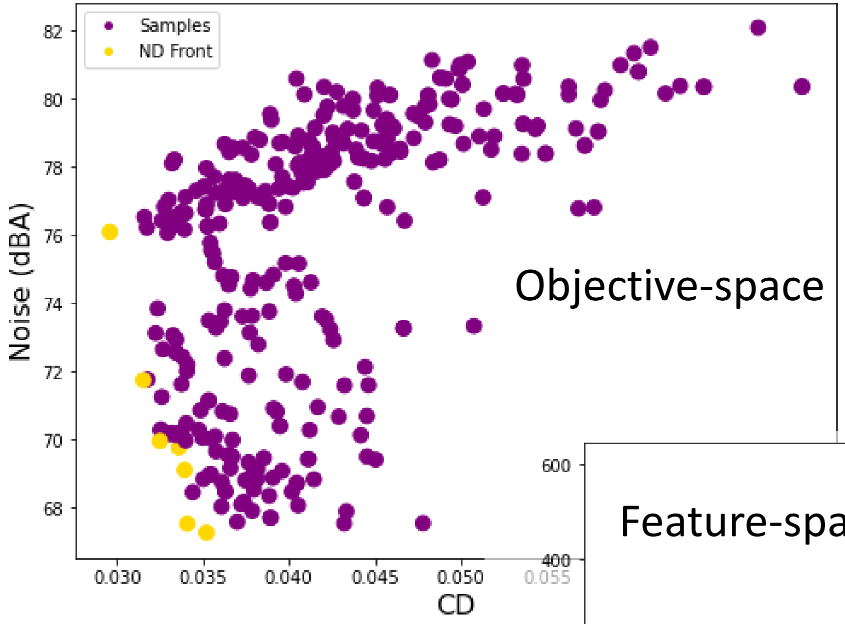
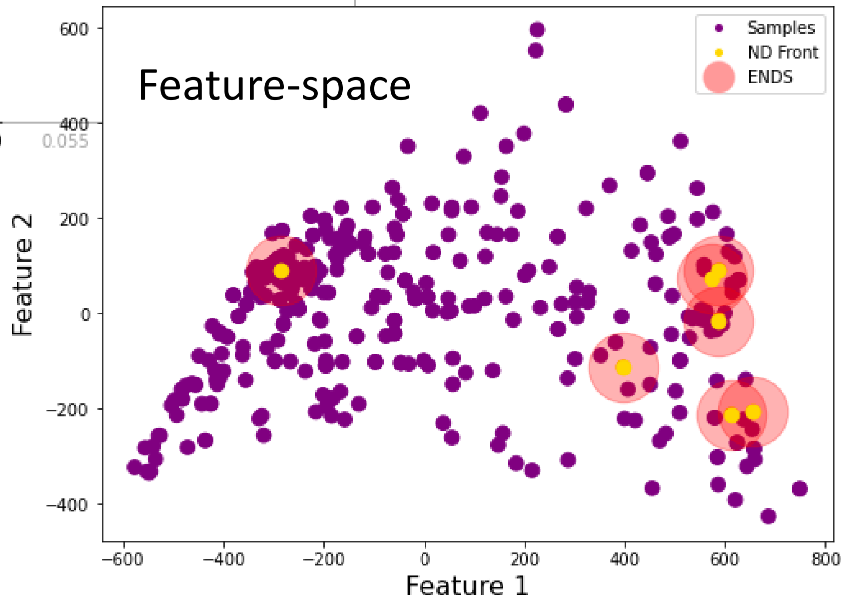
Kriging Believer and Optimizers

- EHVI produces **single candidate** samples per update – slow!
- Kriging Believer (KB) with *5 updates per batch* is implemented to accelerate sample generation & evaluation
- To maximize EHVI we compare **2 optimizers**:
  - **L-BFGS-B** from SciPy with 5 different starting points for hyperparameter optimisation
  - **GA** with localised search near ND front points (GA + ENDS)



Effective Non-Dominant Sampling (ENDS)



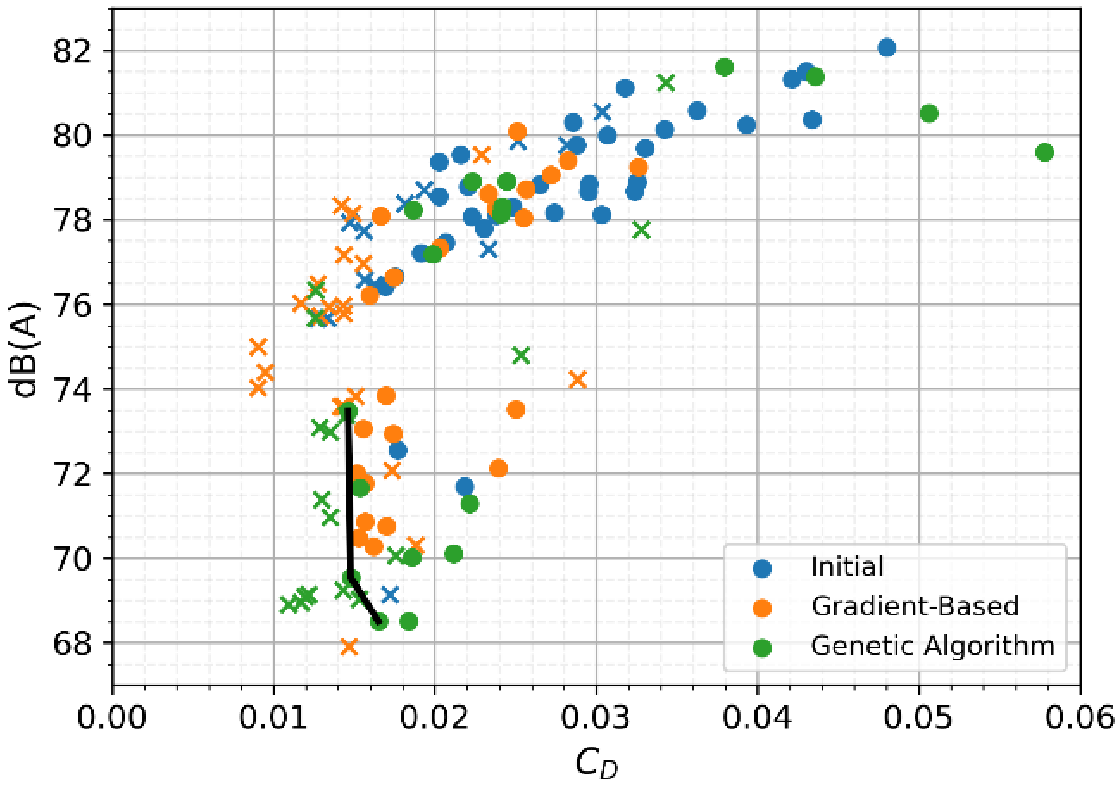


N.B. Feature dimensions reduced for visualisation

- EHVI performs poorly with saturated regions on the ND front
- ENDS is our approach to *localize the EHVI search* of the next candidate
- In place initialising random sampling across the design space
  - Generate normal distributions around each of the ND points
  - Each ND point is used as the mean of each distribution
  - Standard deviation of the distribution is defined as 0.2 of the initial dataset's standard deviation for each design variable

Objective-Space Results (Noise vs Drag)

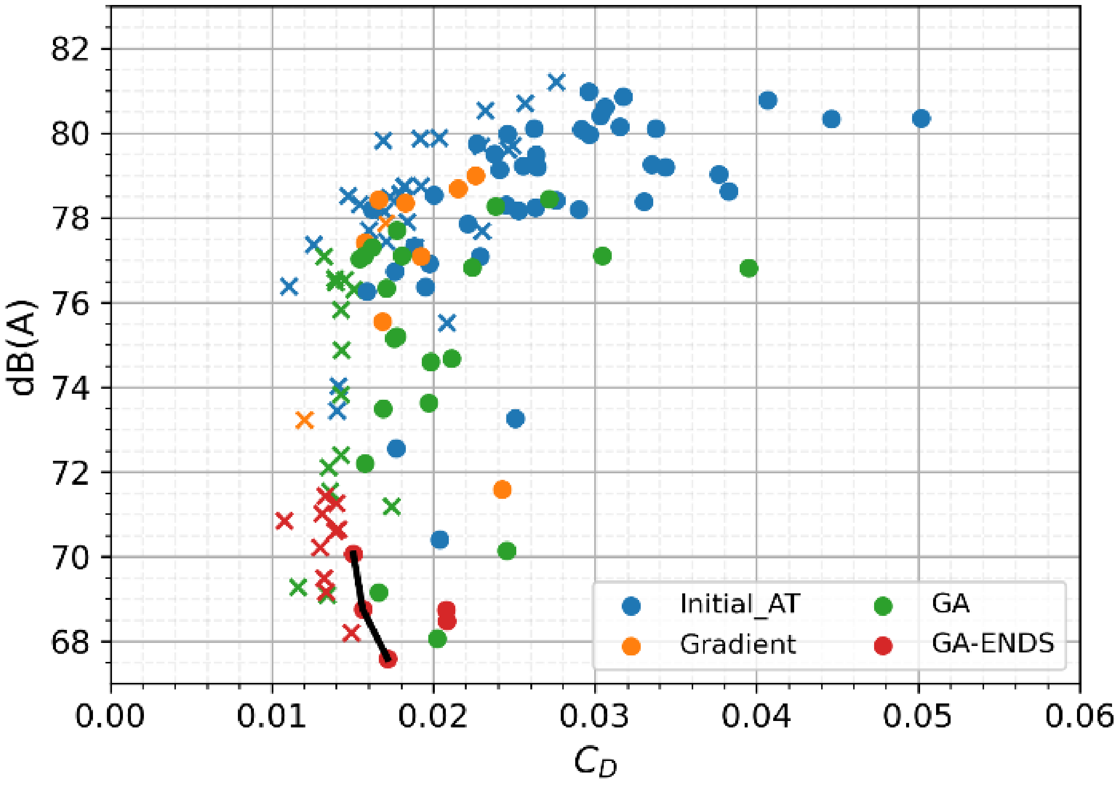
6-variable dataset



Gradient: Initial → 13 updates

GA: (Initial + gradient-based sols. up to 6th update) → 7 updates

11-variable dataset

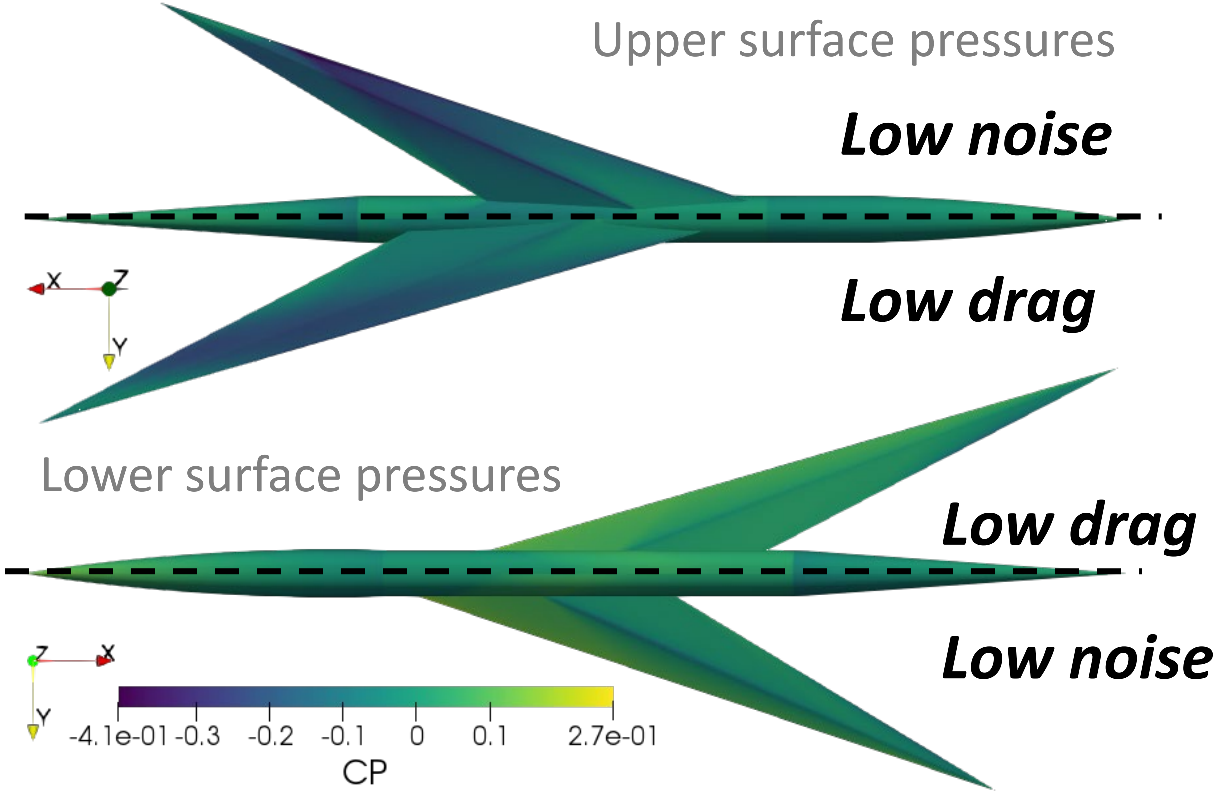


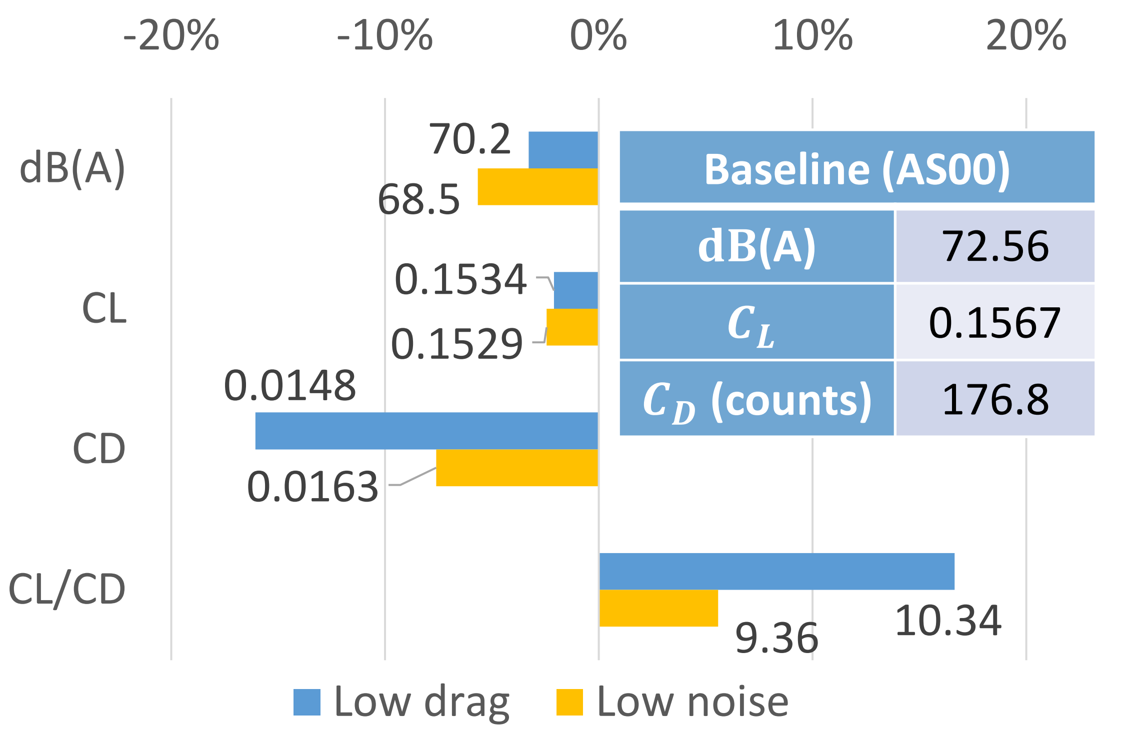
Gradient: Initial → 2 updates

GA: (Initial + gradient-based sols.) → 8 updates

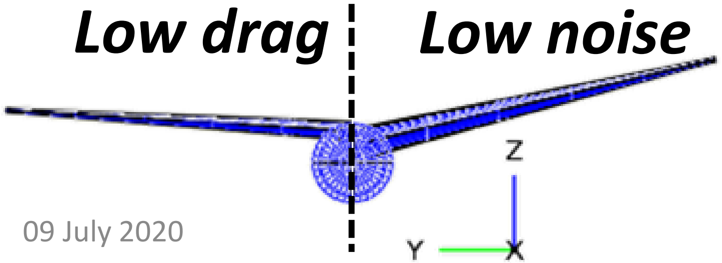
GA-ENDS: (All previous) → 2 updates

6-Variable Results





Baseline (AS00)	
dB(A)	72.56
C_L	0.1567
C_D (counts)	176.8



Common Points
- High spans (low chord)
- High sweeps
- Positive dihedral
- High fuselage position

For Low Drag:
- Thinner wing
- More sweep
- Aftward position

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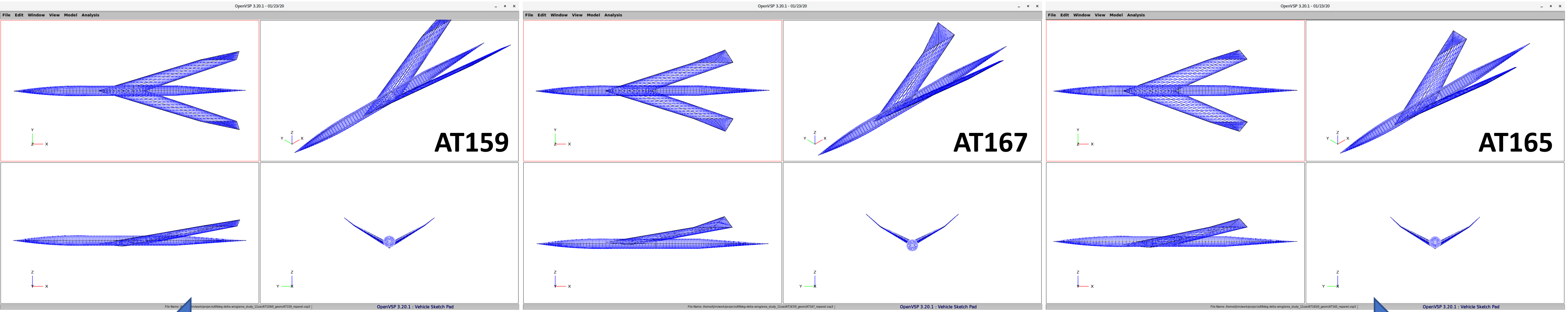
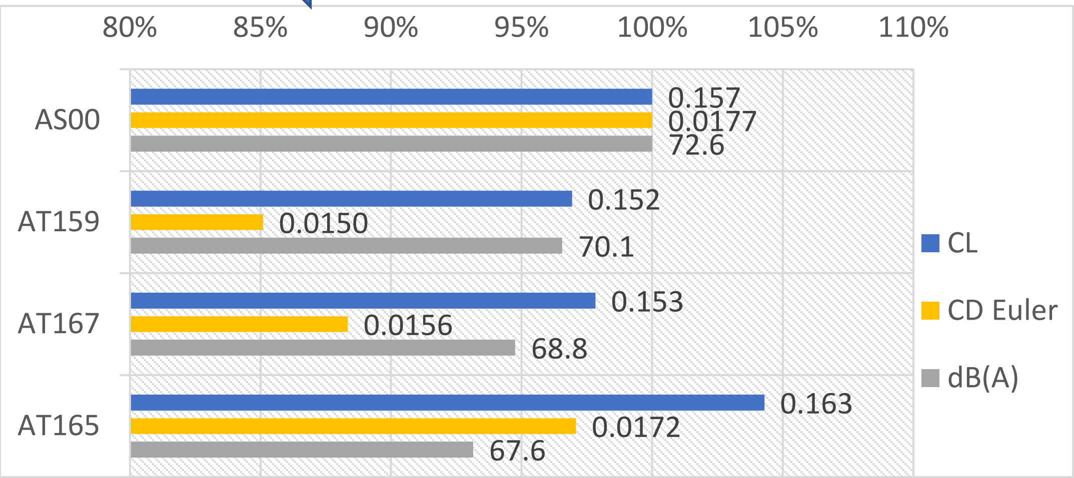
11-Variable Results (GA-ENDS)



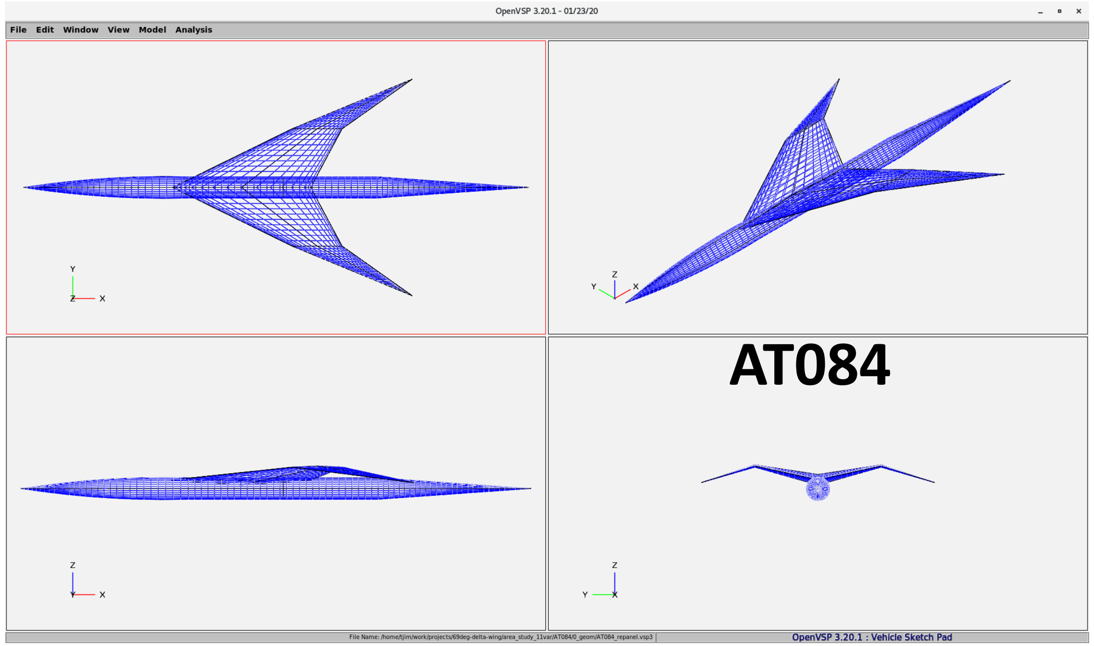
Decreasing Drag | Decreasing Noise

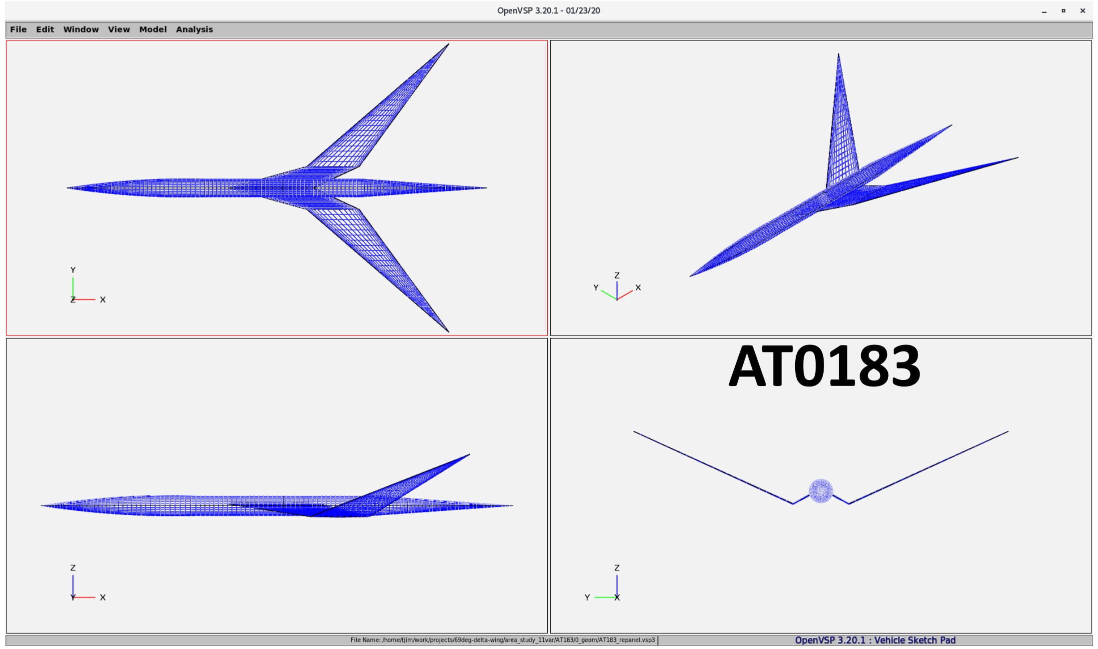


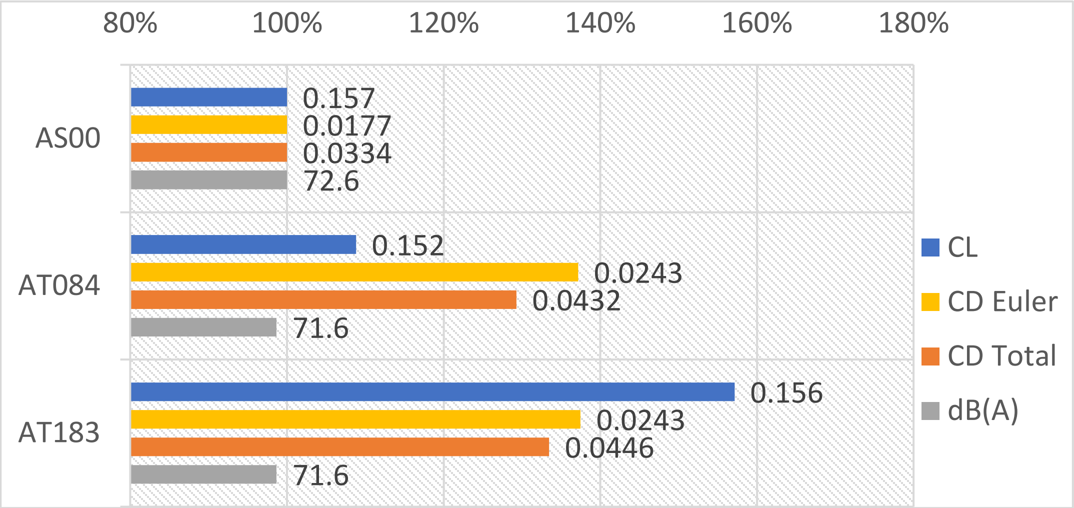
Common Points

- High spans (low chords)
- High sweeps
    - Leading and trailing edges
- Small outboard sections
- Section thicknesses
    - Thicker inboard, thinner outboard
- Positive dihedral

Design Diversity

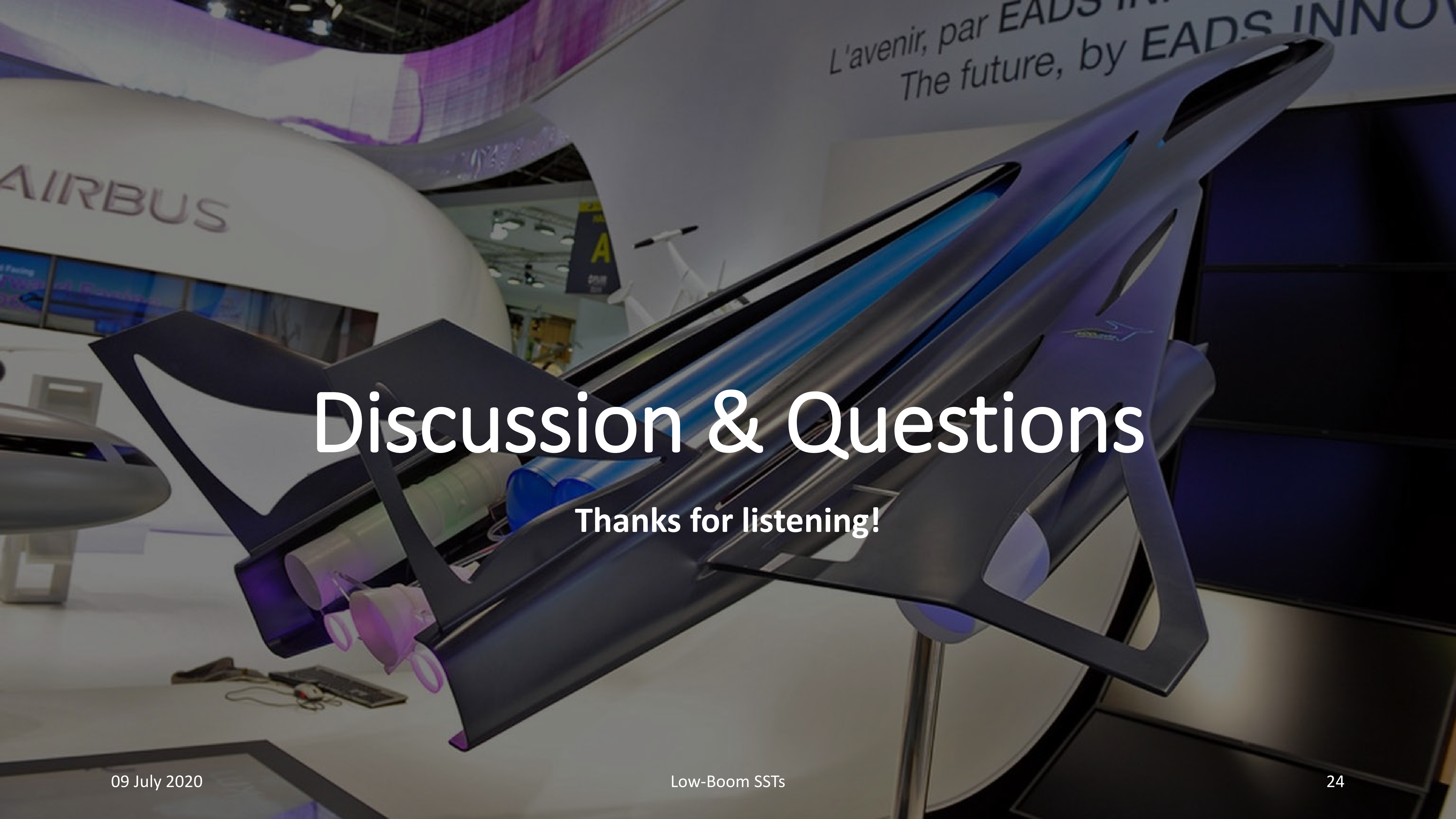






Concluding Points

- Constrained EHVI BO + Kriging Believer was used to optimize a supersonic wing planform
  - Using a gradient-based solver and genetic algorithm
  - For 6-var and 11-var parameterisations
- The GA-based solver could find solutions which gradient-based solver could not
  - At a time penalty (~10x)
  - ***Caution*** – GA results continued the search from some of the gradient-based results
- 6-var vs 11-var parameterisation
  - 6-var: ND points were probably limited by parameterisation
  - 11-var: Diverse set of designs sampled but, the ND front was represented by a small set of similar solutions
- Further work
  - Testing of ENDS for more complex datasets
  - Include an estimate of skin friction drag
  - Drag reduction driving thin wings and high spans
    - Stricter wing-span constraints
  - Explore further from the undertrack



Discussion & Questions

Thanks for listening!